What's Your Strategy?

Belief Formation in Games with Imperfect Feedback

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OSU Theory/Experimental Reading Group

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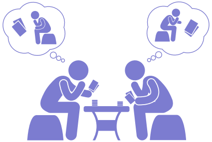
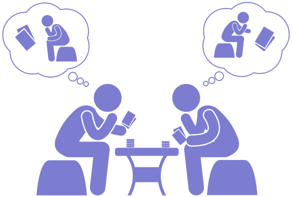
New idea welcoming comments



What we do

Question How do players form beliefs about opponents' strategies when they observe imperfect feedback?

Answer Have lab subjects play a simplified poker game and elicit their beliefs



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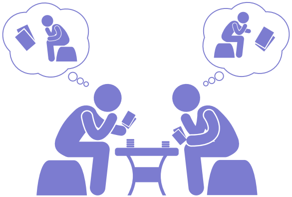


Motivation

Testing the consequences of relaxing a standard assumption

In Nash equilibrium (**NE**, for simultaneous-move games) or perfect Bayesian equilibrium (**PBE**, for extensive-form games) players know opponents' strategies and best respond to them

- **A "learning" interpretation:** Players see the precise outcomes after games end, and by observing many games, they learn others' strategies

⇒ **Q:** how do players form beliefs about others' strategies when game outcomes are imperfectly observed?

  - e.g., In Poker, players don't get to see an opponent's hand when the opponent folds or win by everyone else folding

A simple 2-player game of poker

"Hand" A draw from a deck of 3 cards: **H**igh, **M**iddle, and **L**ow

There are 6 **states**: HM, HL, MH, ML, LH, LM.

Game tree There are at most 2 stages of bets. Game stops if anyone folds.

0. Each player receives one random hand without replacement
1. P1 bets or folds → P2 bets or folds →
2. P1 bets or folds → P2 bets or folds

Payoffs At start, both players put $1 in the pot. Each bet costs additional $1.

The winner takes all $$ in the pot.

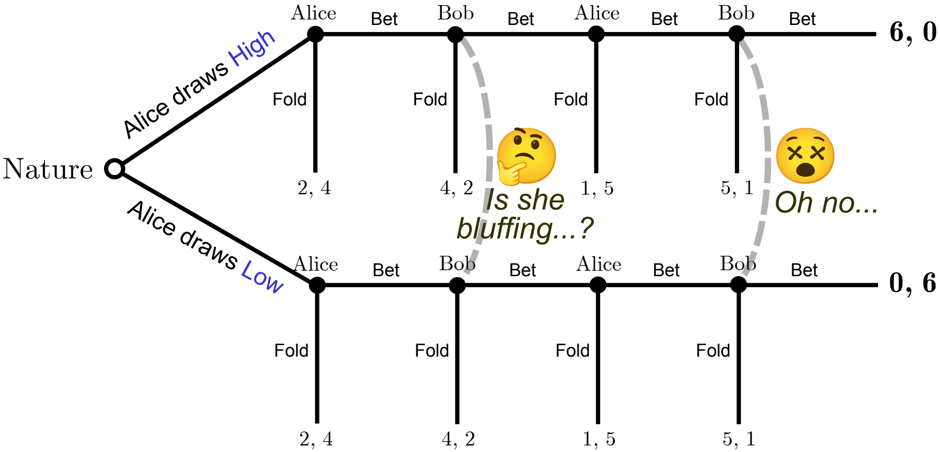
Information & feedback Each player knows that all 6 states are ex-ante equally likely.

Each player learns one's own hand in Stage 0.

Each player learns the other's hand only if everyone bets until the end.

Illustration

A "subgame" from P2's perspective when he draws Middle



Hypotheses

Based on Sungmin's JMP (Park, 2024)

Definition The simple poker game

- has perfect feedback if players observe the state at the end, and
- has imperfect feedback if players observe the state only when all players bet until the end.

Hypothesis When players face imperfect feedback,

1. **(strategies)** they play more extreme strategies (closer to pure strategies) than they do under perfect feedback, and
2. **(beliefs)** they believe their opponents' strategies are less extreme (more mixing) than they really are.

Experimental design

1. Randomly assign each subject permanently to one of four groups:
    - Control Alice, Control Bob, Treated Alice, and Treated Bob
2. In Round 1, randomly match Control Alice to Control Bob and Treated Alice to Treated Bob. Let the control and treated pairs play the game.
    - Give the Control pairs perfect feedback from games played by all Control pairs.
    - Give the Treated pairs imperfect feedback from games played by all Treated pairs.
3. Repeat **Step 2** for Rounds 2–30. Elicit players' beliefs about opponents' strategies after every 5 rounds.
4. Pay each subject based on their performance in a randomly selected round.

Illustration of Perfect feedback

Game outcomes in previous rounds

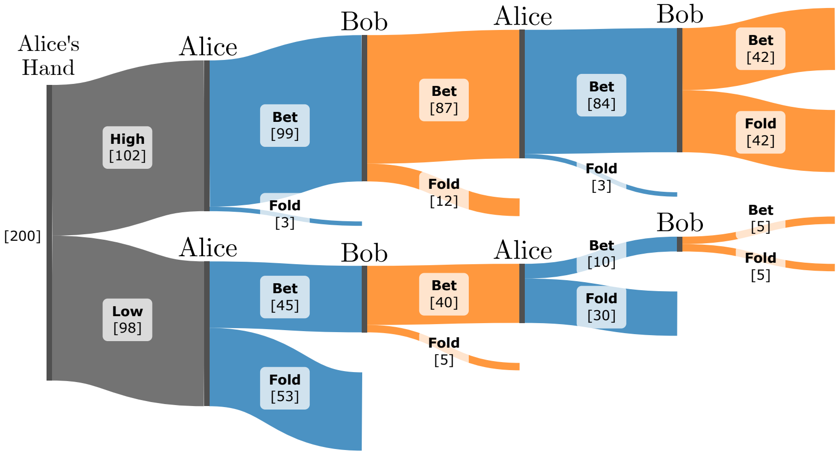
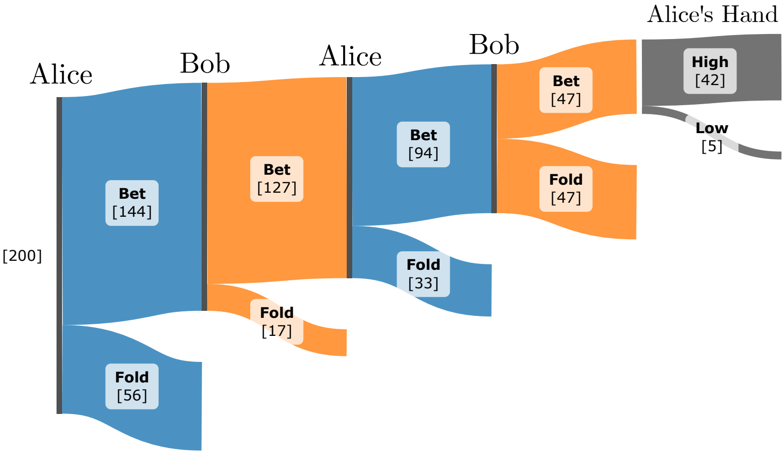


Illustration of Imperfect feedback

Game outcomes from previous rounds



Theoretical background

Let $S = \{HM, HL, MH, ML, LH, LM\}$. Let $A_i = \{B\text{et}, F\text{old}\}$ for all i.

Let $A = \{F, BF, BBF, BBBF, BBBB\}$. Let $\Omega = S \times A$.

Let $\mathcal{I}_i$ denote Player i's collection of information sets.

Definition

- A **strategy** is $\sigma_i : \mathcal{I}_i \to \Delta(A_i)$, representing objective prob. over one's moves
- A **belief** is $\beta_i : \mathcal{I}_{-i} \to \Delta(A_{-i})$, representing subjective prob. over other's moves
- $\mathbf{p}(\sigma_i, \beta_i)$ is the vector of subjective prob. over Ω generated by (σ_i, β_i).
- An **observational structure** is a matrix C with $|\Omega| = 30$ columns
- Given a strategy profile $\sigma = (\sigma_i, \sigma_{-i})$, a belief is **observation-consistent** if

$$C\mathbf{p}(\sigma_i, \beta_i) = C\mathbf{p}(\sigma_i, \sigma_{-i}).$$

Perfect and imperfect observational (feedback) structures

Perfect structure C is the identity matrix I_{30}.

Imperfect structure $C = \widetilde{C}$, a 10×30 matrix where

- 4 rows respresent $P(F), P(BF), P(BBF), P(BBBF)$, and
- 6 rows represent $P(BBBB, s)$ for all states $s \in S$.

State (s): HM (# of Bets (a): 0 1 2 3 4), HL (# of Bets (a): 0 1 2 3 4), MH (# of Bets (a): 0)

$$\widetilde{C} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdots \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdots \\ \hline \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \\ & & & & \vdots & & & & & & & \ddots \end{bmatrix}$$

Equilibrium: Predicted strategies and beliefs

for games with any observation/feedback structure. From Sungmin's JMP

Definition

Given an observational structure C, a triple (σ, β, μ) is a **MaxEnt OCE Equilibrium (MOE)*** if it satisfies the following for every player i:

1. Given (β_i, μ_i), the strategy σ_i is (subjectively) sequentially rational
2. Given σ, the belief β_i maximizes the Shannon entropy of $\mathbf{p}(\sigma_i, \beta_i)$ among observation-consistent beliefs, and
3. Given (σ_i, β_i), the posterior function μ_i is Bayes-consistent

* Maximum-entropy (MaxEnt) observation-consistent expectations (OCE) equilibrium

Remark. Under perfect observational structure ($C = I_{30}$), MOE is equivalent to perfect Bayesian equilibrium (PBE).

1. "Perspectives" for testing players' **strategies**

A perspective is a pair of null and alternative hypotheses (Fay and Proschan, 2010)

1(a). Do Control players play as predicted by PBE?

$$H_0 : \mathbb{E}_{\text{Control}}\big[\mathbf{p}(\sigma)\big] = \mathbf{p}(\sigma^{\text{PBE}}), \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

1(b). Do Treated players play as predicted by MOE?

$$H_0 : \mathbb{E}_{\text{Treated}}\big[\mathbf{p}(\sigma)\big] = \mathbf{p}(\sigma^{\text{MOE}}), \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

1. "Perspectives" for testing players' **strategies**

A perspective is a pair of null and alternative hypotheses (Fay and Proschan, 2010)

1(c). Do Control and Treated players play differently?

$$H_0 : \mathbb{E}_{\text{Control}}\big[\mathbf{p}(\sigma)\big] = \mathbb{E}_{\text{Treated}}\big[\mathbf{p}(\sigma)\big], \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

1(d). (Weaker test) Do Control and Treated players play at similar entropy?

$$H_0 : \mathbb{E}_{\text{Control}}\big[H(\mathbf{p}(\sigma))\big] = \mathbb{E}_{\text{Treated}}\big[H(\mathbf{p}(\sigma))\big], \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

2. "Perspectives" for testing players' **beliefs**

2(a). Are Control players' beliefs correct?

$$H_0 : \mathbb{E}_{\text{Control}}\left[\mathbf{p}(\sigma_i, \tilde{\beta}_i) - \mathbf{p}(\sigma_i, \sigma_{-i})\right] = \mathbf{0}, \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

2(b). Are Treated players' beliefs observation-consistent?

$$H_0 : \mathbb{E}_{\text{Treated}}\left[\widetilde{C}\mathbf{p}(\sigma_i, \tilde{\beta}_i) - \widetilde{C}\mathbf{p}(\sigma_i, \sigma_{-i})\right] = \mathbf{0}, \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

2(c). Are Treated players' beliefs MaxEnt observation-consistent?

$$H_0 : \mathbb{E}_{\text{Treated}}\left[\mathbf{p}(\sigma_i, \tilde{\beta}_i) - \mathbf{p}(\sigma_i, \beta_i^*(\sigma))\right] = \mathbf{0}, \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

3. "Perspectives" for testing convergence of strategies

3(a). Do Control players' strategies converge?

$$H_0 : \mathbb{E}_{\text{Control, Rounds } X}\left[\mathbf{p}(\sigma)\right] = \mathbb{E}_{\text{Control, Rounds } X'}\left[\mathbf{p}(\sigma)\right], \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

3(b). Do Treated players' strategies converge?

$$H_0 : \mathbb{E}_{\text{Treated, Rounds } X}\left[\mathbf{p}(\sigma)\right] = \mathbb{E}_{\text{Treated, Rounds } X'}\left[\mathbf{p}(\sigma)\right], \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

3(c). Do Control and Treated players' strategies converge at the same rate?

$$H_0 : \mathbb{E}_{\text{Control, Rounds } X}\left[\mathbf{p}(\sigma)\right] - \mathbb{E}_{\text{Control, Rounds } X'}\left[\mathbf{p}(\sigma)\right]$$
$$= \mathbb{E}_{\text{Treated, Rounds } X}\left[\mathbf{p}(\sigma)\right] - \mathbb{E}_{\text{Treated, Rounds } X'}\left[\mathbf{p}(\sigma)\right], \text{ and}$$

$$H_a : H_0 \text{ is false.}$$

Literature

Closest paper: Huck, Jehiel and Rutter (2011)

- Nature chooses a game state A or B. Pairs of players see the game type and play the same simultaneous-move game form with different payoffs.
- After each round, Treated players see the actions chosen by all Treated players but not the game type. Control players see everything.
- They find that Treated players' strategies are aligned with ABEE*. Control players' strategies are aligned with NE.

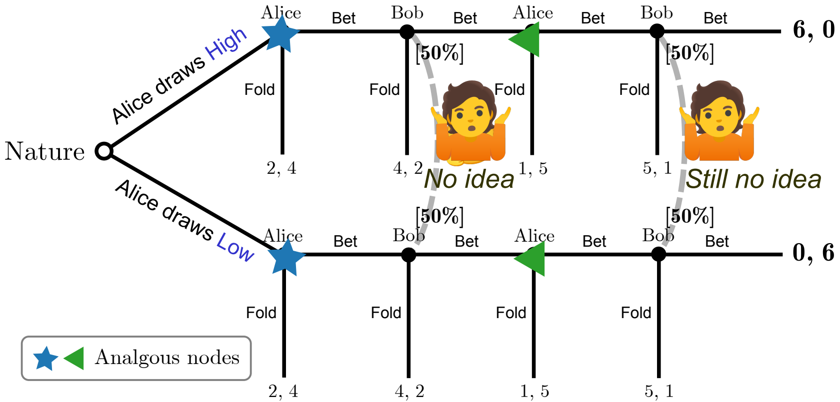
  * Analogy-Based Expectation Equilibrium (Jehiel, 2005): Each player believes that opponents behaves the same in "analogous" nodes.

Remark. In their game and observational structure, ABEE $\Leftrightarrow$ MOE.

Contribution. Our experiment tests the impact of imperfect feedback in a game where ABEE $\not\Leftrightarrow$ MOE and ABEE is not so attractive.

Consequences of ABEE in the simple poker game

Bob has no clue about Alice's hand even after she keeps betting



Intuition. ABEE assumes that each player believes the opponent behaves in the same way in analogous nodes.

(Expected) Takeaways from our experiment

When there is imperfect feedback in games,

- opponents' strategies are less transparent,
- so players take more extreme strategies
    - e.g., less frequent bluffing

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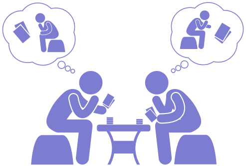

Appendix

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