

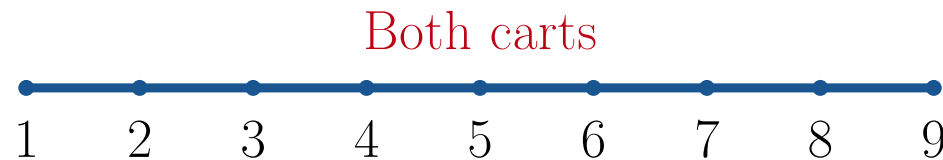
Chapter 8

Location, Partnership, and Social Unrest



Location Choice

Ben and Jerry choose locations $1, \dots, 9$ on a path. Each section has 100 customers, who buy from the closer cart; ties split equally. Prices and margins are fixed, so each cart maximizes its customer count.



Iterated elimination of strictly dominated strategies (IESDS) removes 1, 9, then 2, 8, then 3, 7, then 4, 6. Both choose 5 and serve 450 customers each. All customers stay in place throughout the deletions.

Discussion: A Fixed Wage

Keep the same nine locations and 900 customers. Suppose each cart operator receives a fixed wage and dislikes serving customers. Each now wants to *minimize* the number served.

Compare location 1 with locations 2 through 5 against every possible rival location. What does symmetry imply for location 9?

Which locations survive iterated strict dominance? Does the prediction of both carts at the center still hold?

Answer: The Endpoints Survive

Location 1 serves strictly fewer customers than each location 2, 3, 4, 5, whatever the rival chooses. Location 9 similarly dominates 5, 6, 7, 8.

After deleting the interior locations, either endpoint serves 450 customers against either rival endpoint. Both choices are best responses, so

$$R_1 = R_2 = \{1, 9\}.$$

Changing how effort is rewarded changes the strategic incentives, even though the geography and customer behavior are unchanged.

A Partnership

Two partners run a small design studio. Partner 1 chooses effort x and partner 2 chooses effort y , simultaneously, with $x, y \in [0, 4]$.

The firm's gross profit is

$$\pi = 4(x + y + cxy), \quad 0 < c < \frac{1}{4}.$$

The partners share gross profit equally. Their personal effort costs are x^2 and y^2 . Effort cannot be verified by a court, so their agreement cannot enforce particular effort levels.

Private Payoffs and Joint Surplus

Each partner receives half of gross profit and bears her own effort cost:

$$\begin{aligned}u_1(x, y) &= 2(x + y + cxy) - x^2, \\u_2(x, y) &= 2(x + y + cxy) - y^2.\end{aligned}$$

The partners' joint surplus is

$$W(x, y) = u_1 + u_2 = 4(x + y + cxy) - x^2 - y^2.$$

An extra unit of gross profit benefits both partners. The partner supplying the effort receives only half of that benefit.

Best Response to a Belief

Let $\bar{y}$ be partner 1's expected y , and $\bar{x}$ partner 2's expected x . Partner 1's expected payoff is

$$2(x + \bar{y} + cx\bar{y}) - x^2.$$

The first derivative is $2 + 2c\bar{y} - 2x$ and the second derivative is -2 . Thus, the unique best response is

$$BR_1(\bar{y}) = 1 + c\bar{y}, \quad BR_2(\bar{x}) = 1 + c\bar{x}.$$

These choices lie in $[1, 1+4c] \subset [0, 4]$, so no boundary adjustment is needed. Only the mean of the belief matters because payoffs are linear in the other partner's effort.

Discussion: Uncertain Effort

Let $c = 1/5$. Partner 1 believes that partner 2 chooses $y = 0$ or $y = 4$, each with probability $1/2$.

Find partner 1's optimal effort. Would her choice differ if she expected $y = 2$ with probability one?

Does either belief justify choosing effort $x = 3$?

Answer: The Mean Determines Effort

Both beliefs have mean $\bar{y} = 2$. Therefore, both imply

$$BR_1(2) = 1 + \frac{1}{5}(2) = \frac{7}{5} = 1.4.$$

For either belief, expected payoff as a function of x is

$$4 + \frac{14}{5}x - x^2.$$

This strictly concave expression is maximized at 1.4, not at 3. In fact, 3 is never a best response to any belief over $[0, 4]$, since $1 + \bar{y}/5 \leq 1.8$.

Strategic Complementarity

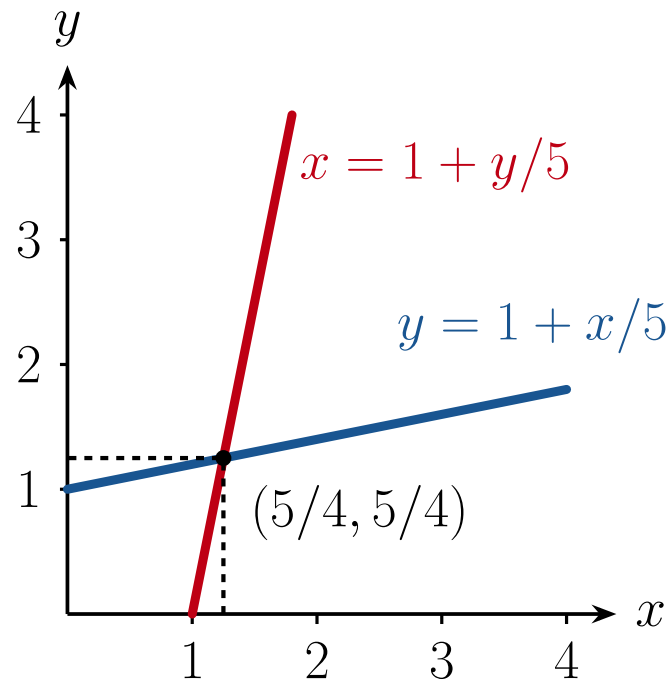
Definition. Actions are **strategic complements** when a higher action by one player raises the other player's incentive to choose a higher action.

For the partnership,

$$\frac{\partial^2 u_1}{\partial x \partial y} = \frac{\partial^2 u_2}{\partial y \partial x} = 2c > 0.$$

A higher expected effort by the other partner raises one's own best response. A positive effect on the other partner's payoff alone does not establish strategic complementarity; the effect on marginal incentives matters.

The Best-Response Curves



The example uses $c = 1/5$. The red curve expresses x as a function of y ; with y on the vertical axis, its plotted slope is 5.

First-Round Restrictions

Since the other partner's effort lies in $[0, 4]$, its expected value also lies in $[0, 4]$. Every best response therefore lies in

$$R_i^1 = [1, 1 + 4c].$$

Every effort in this interval is optimal against some certain opponent effort. An effort below 1 is strictly dominated by 1; an effort above $1 + 4c$ is strictly dominated by $1 + 4c$.

For $c = 1/5$, the surviving interval is $[1, 1.8]$ for each partner.

Discussion: Restrict Beliefs Again

Suppose $c = 1/5$ and the game and rationality are common knowledge. The first-round surviving interval is $[1, 1.8]$.

What are the smallest and largest possible means of a belief supported on this interval? Apply $BR_i(m) = 1 + m/5$ to these two means.

Find the interval surviving round 2 and then round 3. Are these intervals already singletons?

Answer: Shrinking Intervals

The second-round interval is

$$R_i^2 = [1 + \frac{1}{5}(1), 1 + \frac{1}{5}(1.8)] = [1.2, 1.36].$$

The third-round interval is

$$R_i^3 = [1 + \frac{1}{5}(1.2), 1 + \frac{1}{5}(1.36)] = [1.24, 1.272].$$

Each interval still contains many effort levels. The bounds restrict beliefs about the other partner, which then restrict optimal effort. The rounds describe reasoning, not a sequence of actual effort choices.

A Unique Rationalizable Effort

Write $R_i^k = [L_k, U_k]$, with $L_0 = 0$ and $U_0 = 4$. The bounds satisfy

$$L_{k+1} = 1 + cL_k, \quad U_{k+1} = 1 + cU_k.$$

Let $r = 1/(1 - c)$. Then

$$L_k = r - rc^k, \quad U_k = r + (4 - r)c^k.$$

Because $0 < c < 1/4$, both bounds converge to r . Thus,

$$\bigcap_{k=0}^{\infty} R_i^k = \{r\}, \quad x^* = y^* = \frac{1}{1 - c}.$$

Why the Bounds Converge

The surviving interval has width

$$U_k - L_k = 4c^k.$$

Every round multiplies the width by $c < 1$. It remains positive after any finite number of rounds but converges to zero.

This contraction yields uniqueness in the partnership model. Strategic complementarity alone does not imply a unique rationalizable outcome; best responses can reinforce more than one pattern of behavior.

Discussion: Jointly Optimal Effort

The partners' total payoff is

$$W(x, y) = 4(x + y + cxy) - x^2 - y^2.$$

Find the effort pair that maximizes W . Compare it with the rationalizable effort $x^* = y^* = 1/(1 - c)$.

Which benefit does an individual partner leave out when choosing effort?

Answer: Both Partners Supply Too Little

The first-order conditions for joint surplus are

$$4 + 4cy - 2x = 0, \quad 4 + 4cx - 2y = 0.$$

They imply

$$\hat{x} = \hat{y} = \frac{2}{1 - 2c} > \frac{1}{1 - c} = x^* = y^*.$$

The solution belongs to $[0, 4]$ for each partner and is the unique feasible joint optimum. Each partner ignores the extra profit her effort gives the other partner.

The Cost of Underproviding Effort

For $c = 1/5$, the effort and payoff comparisons are

	Rationalizable	Joint optimum
Each partner's effort	$5/4$	$10/3$
Each partner's payoff	$65/16 \approx 4.06$	$20/3 \approx 6.67$

Both partners would gain at the joint optimum. But if partner 2 chooses $10/3$, partner 1's best response is $1 + (10/3)/5 = 5/3$, not $10/3$. An unenforced promise to supply the jointly optimal effort is not enough.

A Model of Social Unrest

A population of citizens is indexed uniformly by $i \in [0, 1]$. Each chooses simultaneously whether to protest (P) or stay home (H).

Let $z \in [0, 1]$ be the fraction protesting, and let $\alpha \geq 0$ measure preference for or dedication to protesting. The payoffs are

$$u_i(H, z) = 4z - 2, \quad u_i(P, z) = 8z - 4 + \alpha i.$$

A higher index means greater dedication to protesting. Each individual has zero mass, so one person's choice does not change z .

The Incentive to Participate

The payoff gain from protesting is

$$\Delta_i(z) = u_i(P, z) - u_i(H, z) = 4z - 2 + \alpha i.$$

A citizen strictly prefers P if $\Delta_i(z) > 0$, strictly prefers H if $\Delta_i(z) < 0$, and is indifferent at equality.

For $\alpha > 0$, the cutoff is $\bar{i} = (2 - 4z)/\alpha$. If $\bar{i} \in [0, 1]$, citizen $i = \bar{i}$ is indifferent. Citizens with $i > \bar{i}$ strictly prefer protesting; those with $i < \bar{i}$ strictly prefer staying home. If $\bar{i} < 0$, everyone strictly prefers protesting; if $\bar{i} > 1$, everyone strictly prefers staying home.

Higher participation makes protesting more attractive to every citizen. If z is uncertain, the same comparison uses its expected value.

Discussion: Can Everyone Stay Home?

Let $\alpha = 1$. Consider each citizen's incentives under two beliefs:

- Nobody else protests, so $z = 0$.
- Everybody else protests, so $z = 1$.

Can either action be eliminated for any citizen? Is everyone staying home consistent with common knowledge of rationality?

Compare everyone's payoffs when all stay home with those when all protest.

Answer: Strategic Uncertainty Persists

With $\alpha = 1$, the payoff gain is $\Delta_i(z) = 4z - 2 + i$. At $z = 0$, it is negative for every citizen. At $z = 1$, it is positive for every citizen.

Both actions have supporting beliefs for everyone, so no deletion starts. Every profile is rationalizable, including everyone staying home.

All staying home gives each citizen -2 . All protesting gives citizen i the payoff $4 + i$. Everyone prefers the latter outcome.

Discussion: A Participation Cascade

Let $\alpha = 3$, so the gain from protesting is $4z - 2 + 3i$.

1. Which citizens strictly prefer P even at $z = 0$?
2. What lower bound on participation follows? Which additional citizens must then prefer P ?
3. Use the new lower bound to determine whether anyone can still choose H .

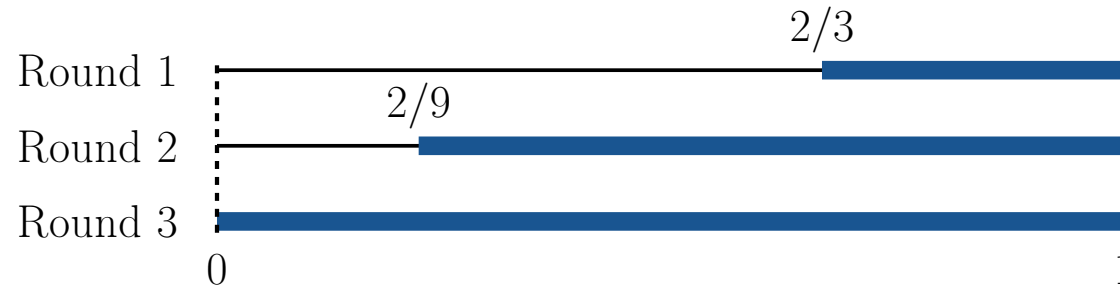
Keep strict inequalities distinct from indifference at the cutoff.

Answer: Three Rounds

1. At $z = 0$, protesting is strictly better for $i > 2/3$. These citizens have mass $1/3$, so $z \geq 1/3$.
2. At $z = 1/3$, the gain is $3i - 2/3$. Thus, $i > 2/9$ must protest, implying $z \geq 7/9$.
3. At $z = 7/9$, even citizen $i = 0$ gains $10/9 > 0$ from protesting. Everyone must protest.

A single citizen at a cutoff has zero mass. Including or excluding that indifferent citizen does not change the participation bound.

Participation Bounds



Blue marks citizens who must protest when $\alpha = 3$. The guaranteed participation fractions are $1/3$, $7/9$, and 1.

These are deductions before simultaneous play. Nobody needs to observe an actual protest in an earlier round.

Dedication and the Initial Participants

If $0 \leq \alpha \leq 2$, every citizen can justify staying home by believing $z = 0$, and protesting by believing $z = 1$. No action is strictly dominated. At $\alpha = 2$, citizen $i = 1$ is indifferent when $z = 0$.

If $\alpha > 2$, citizens with $i > 2/\alpha$ strictly prefer to protest even when nobody else does. Iterated reasoning yields universal participation.

The conclusion uses common knowledge of payoffs and rationality. The model isolates an incentive mechanism; it does not establish that any particular real protest must occur.

Takeaways

Incentives and beliefs jointly shape rationalizable outcomes.

- Location choices depend on how serving customers is rewarded. Fixed prices and customer-count payoffs lead both carts to the center.
- In a partnership, complementary effort can yield a unique rationalizable outcome while leaving both partners worse off than at their joint optimum.
- Participation incentives depend on beliefs about how many others join. Sufficiently dedicated citizens can initiate a cascade of reasoning.