

National Taiwan University
Department of Economics
ECON 4037: Game Theory
Homework 2: Chapters 5–7

Due 2:20 PM on Wednesday, October 7, 2026
You may submit online or hand in a handwritten copy.

Show your work. You may discuss these problems with classmates or use generative AI tools while working on them, but you must write your own answers and be able to explain and defend everything you submit (see the syllabus's Course Requirements).

Throughout, “dominated” means *strictly* dominated, possibly by a mixed strategy. Include ties when reporting best-response sets. Payoffs in each matrix or tree are listed as (player 1, player 2).

1. **Rationality and common knowledge.**

- (a) Mei has NT\$600 and chooses either to keep it all or to give NT\$300 to a friend. The friend has no decision to make. Mei's utility is $u = m_M + 2m_F$, where m_M and m_F are the amounts of this NT\$600 received by Mei and her friend. Which choice is rational? Would choosing to give money show that Mei is not maximizing her payoff? Explain the distinction between money and utility.
- (b) Two students each privately receive and read a reliable message that an event is canceled. Neither is told whether the other received a message. Both know the cancellation, but is it necessarily common knowledge? Identify a level of knowledge that may be missing. Contrast this with a reliable public announcement that both observe, under common knowledge that both observe and understand it.
- (c) Jun chooses between NT\$100 for sure and a lottery paying NT\$200 with probability $1/2$ and zero otherwise. He maximizes expected utility with $u(m) = \sqrt{m}$, where m is the amount received. Compare expected money and expected utility for the two options. Is rejecting the lottery rational?

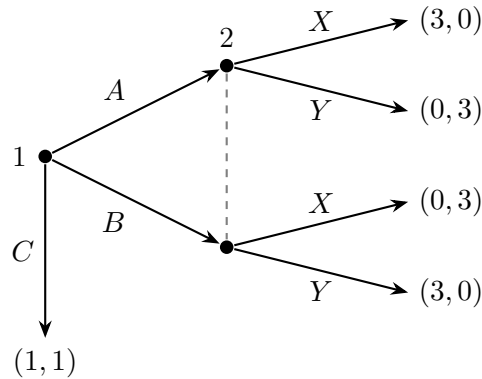
2. **Rock–paper–scissors.** Each player simultaneously chooses Rock (R), Paper (P), or Scissors (S). Rock beats Scissors, Scissors beats Paper, and Paper beats Rock. A win pays 1, a loss pays -1 , and a tie pays 0.

- (a) Write the normal-form payoff matrix, using the order (R, P, S) for both players.
- (b) For each belief θ_2 below, calculate player 1's expected payoff from each pure strategy and report the set of pure best responses $BR_1(\theta_2)$:

$$(1, 0, 0), \quad \left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right), \quad \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right), \quad \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

- (c) Which pure strategies are rationalizable for each player? Justify your answer using supporting beliefs.

3. **A safe option?** Player 1 chooses A , B , or C . If she chooses A or B , player 2 chooses X or Y without observing which of those two choices she made. The dashed line marks player 2's information set.



- (a) State both players' pure-strategy sets and write the normal-form matrix.
- (b) Does any pure strategy strictly dominate C ? Find all probabilities q for which playing A with probability q and B with probability $1 - q$ strictly dominates C .
- (c) Let p be player 1's belief that player 2 chooses X . Show directly, by comparing expected payoffs, whether C can ever be a best response. What are the rationalizable strategies for each player?
4. **Iterated dominance.** Consider this game:

	X	Y	Z
U	2, 0	1, 1	4, 2
M	3, 4	1, 2	2, 3
D	1, 3	0, 2	3, 0

- (a) Find each player's rationalizable pure strategies. Record each deletion and show the strict payoff comparisons that justify it.
- (b) To verify that deletion should stop, give a belief over the other player's surviving strategies that makes each remaining strategy a best response.
- (c) Does rationalizability select a unique pure-strategy profile here? State the full set of surviving profiles.
- (d) Identify all Pareto-efficient pure-strategy profiles in the original game (compare only pure profiles). Is every rationalizable profile efficient? Give a payoff comparison to support your answer.
- (e) Can (U, X) be rationalizable even though neither player best responds to the other's actual choice? Give supporting beliefs. How could a nonbinding preplay agreement help the players coordinate without changing payoffs?

5. **A workplace dispute.** An employee chooses Settle or Be tough, while an employer simultaneously chooses Give in or Hire an attorney. Payoffs are utilities, with the employee's payoff listed first. The real number x is common knowledge.

	Give in	Hire an attorney
Settle	1, 2	0, 1
Be tough	3, 0	$x, 1$

- Let p be the employer's belief that the employee will Settle. For which p is Give in a best response? Include the boundary case.
 - Find the rationalizable strategies of both players when $x > 0$, $x = 0$, and $x < 0$. For every strategy you retain, identify a supporting belief; for every deletion, give a strict dominance argument.
 - For which values of x can Give in be rationalized? Explain why the case $x = 0$ cannot be treated in the same way as $x > 0$.
 - Suppose $x > 0$, but the employer does not know whether the employee is rational. Could a rational employer choose Give in? Explain which additional knowledge justified its deletion above.
6. **Ten-player guessing game.** Players $i = 1, \dots, 10$ simultaneously choose integers $s_i \in \{0, 1, \dots, 10\}$. Let

$$a = \frac{s_1 + \dots + s_{10}}{10}, \quad u_i = (a - i - 1)s_i.$$

These are the game's payoffs; there is no prize for being closest to a target. This is a different game from the live two-thirds-of-the-average game.

- Put $T_i = \sum_{j \neq i} s_j$ and express u_i as a function of s_i and T_i . Remember that changing s_i also changes a .
- For $s_i = k \in \{1, \dots, 9\}$, compare choosing k with the mixture that chooses 10 with probability $k/10$ and 0 otherwise. Show that this mixture strictly dominates k for every fixed opponent profile.
- Compare $s_i = 0$ and $s_i = 10$. Starting from the original game, repeatedly remove *all* strictly dominated strategies for all players at each round. Which endpoint is removed for which player in each round? Be careful not to delete a strategy merely because it sometimes ties another.
- State the final rationalizable set for every player. Explain why the many-player correlation issue discussed in lecture does not obstruct your argument here.

Sources: Problems 2–4 and 6 adapt Watson (2013) Ch. 6 Exercises 5–6, and Ch. 7 Exercises 1(b) and 7. Problem 5 adapts Ch. 7 Exercise 2. Problem 1 applies the concepts in Ch. 5.