

National Taiwan University
Department of Economics
ECON 4037: Game Theory
Homework 3: Chapters 8–11

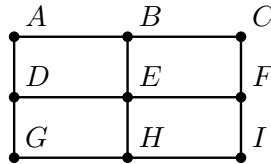
Due 2:20 PM on Wednesday, October 21, 2026

You may submit online or hand in a handwritten copy.

Show your work. You may discuss these problems with classmates or use generative AI tools, but you must write your own answers and be able to explain and defend everything you submit (see the syllabus).

Include all ties in best-response sets. In Problems 1–5, find only *pure-strategy* equilibria; Problem 6 asks for all equilibria.

1. **Location on a grid (Watson Ch. 8, Exercise 1).** Two tea vendors simultaneously choose one of the nine locations shown below. Each location has two customers. Each customer buys from the nearer vendor, generating one unit of profit. Distance is the number of horizontal and vertical steps; diagonal travel is not allowed. Customers equidistant from the vendors split equally between them, including when both vendors choose the same location.



- (a) Compare the profit from A with the profit from E for each of the rival's nine locations. Use symmetry to identify strictly dominated corner locations.
 - (b) After deleting the corners for both players, compare B with E against every surviving rival location. Find both players' rationalizable sets. State the strategy removed after each comparison.
 - (c) Find all Nash equilibria and each vendor's profit in equilibrium. Explain why there cannot be another equilibrium in the original game.
2. **Effort in a partnership.** Two partners choose efforts $x, y \in [0, 4]$ simultaneously. Gross profit is $4(x+y+xy/8)$, shared equally. Their personal effort costs are x^2 and y^2 , respectively.
 - (a) Write each partner's payoff and derive her best response to a belief about the other partner's effort. Check the effort bounds. Are efforts strategic complements? Explain using marginal incentives.
 - (b) Assume common knowledge of the game and rationality. Starting from $[0, 4]$, find the first two rounds of surviving effort intervals. Derive a recursion for their endpoints and find the rationalizable set in the limit.
 - (c) Find all Nash equilibria. On a diagram with x horizontal and y vertical, draw both best-response curves and label their intersection.
 - (d) Find the effort pair maximizing the sum of the partners' payoffs. Show that it is the unique maximizer, calculate each partner's payoff there and in equilibrium, and explain why an unenforced promise to supply the jointly optimal effort is not self-enforcing.

3. **Participation and indifference.** Citizens are indexed uniformly by $i \in [0, 1]$. Each simultaneously chooses Protest (P) or Home (H). If the fraction protesting is z , payoffs are

$$u_i(H, z) = 4z - 2, \quad u_i(P, z) = 8z - 4 + 2i.$$

Each citizen has zero mass, so her own action does not change z .

- Find the payoff gain from protesting. At $z = 0$, which citizens strictly prefer H , and who is indifferent? Can iterated deletion of strictly dominated actions get started? Find each citizen's rationalizable actions, giving supporting beliefs.
 - Find every equilibrium participation fraction z . For an interior fraction, derive the cutoff index and impose consistency between the fraction who protest and the fraction who prefer to protest. State how indifferent citizens can choose at each equilibrium fraction.
 - Compare everyone's payoffs when all stay home with those when all protest. Does a Pareto improvement rule out the all-home equilibrium? Explain using unilateral deviations.
4. **Mutual best responses (Game I: Watson Ch. 9, Exercise 3(b)).** Consider the following two games. Player 1 chooses a row and player 2 chooses a column.

Game I				Game II		
	X	Y	Z		L	R
U	2, 0	1, 1	4, 2	A	3, 2	0, 0
M	3, 4	1, 2	2, 3	B	3, 1	2, 2
D	1, 3	0, 2	3, 0			

- List each player's best responses to every pure opponent strategy in each game. Find all Nash equilibria and identify which are strict.
- In Game I, U and X are rationalizable strategies. Give a belief supporting each, then explain why (U, X) is not an equilibrium. Identify one profitable deviation for each player at that profile.
- In Game II, compare the payoffs at its equilibria. Can an inefficient equilibrium be self-enforcing? Does a nonbinding agreement to play (B, L) make that profile self-enforcing?

5. **Evidence production (Watson Ch. 10, Exercise 6).** A prosecutor (P) and a defendant (D) simultaneously choose expenditures $e_P, e_D \geq 0$ on evidence. If $e_P + e_D > 0$, the probability of conviction is $e_P/(e_P + e_D)$. If both spend zero, the probability of conviction is $1/2$. Conviction gives the prosecutor a benefit of 8 and imposes a loss of 8 on the defendant. Acquittal gives both a benefit of zero. Each pays her or his own expenditure regardless of the verdict and maximizes expected benefit minus expenditure.
- Represent the game in normal form. State the payoff functions both when total expenditure is positive and when both expenditures are zero.
 - Derive each player's best-response correspondence for every possible opponent expenditure. Check nonnegativity, and treat an opponent expenditure of zero separately. Does a best response always exist?
 - Find all pure-strategy Nash equilibria and the probability of acquittal in equilibrium. Check that no equilibrium has a zero expenditure.
 - Calculate both equilibrium payoffs. Is the equilibrium Pareto efficient? Give a specific alternative expenditure pair and compare payoffs. Explain why a joint reduction in expenditures need not be self-enforcing.
6. **Mixed-strategy equilibria (Watson Ch. 11, Exercise 6(d) and (f)).** Consider the following games. Player 1 chooses a row and player 2 chooses a column. Players randomize independently. Probability vectors follow the displayed row or column order.

Game III			Game IV			
	A	B		L	M	R
A	1, 4	2, 0	U	8, 1	0, 2	4, 3
B	0, 8	3, 9	C	3, 1	4, 4	0, 0
			D	5, 0	3, 3	1, 4

- Find all pure-strategy Nash equilibria in both games.
- In Game III, let $p = \Pr_1(A)$ and $q = \Pr_2(A)$. Calculate the expected payoff from each pure action, find every equilibrium involving randomization, and report both players' expected payoffs at each such equilibrium. Explain why no equilibrium has exactly one player randomizing.
- In Game IV, first identify a strictly dominated column. Then let $\sigma_1 = (p, r, 1 - p - r)$ and $\sigma_2 = (0, q, 1 - q)$. Find all equilibria involving randomization, including boundary values of p, r, q . Check every unused action as well as indifference among actions used with positive probability. Report each player's expected payoff along the equilibrium set.
- Explain why equating the payoffs of two actions is necessary when both receive positive probability, but is not by itself sufficient to establish a mixed-strategy equilibrium.